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Quenched disorder effects on deterministic inertia ratchets

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The effect of quenched disorder on the underdamped motion of a periodically driven particle on a ratchet potential is studied. As a consequence of disorder, current reversal and chaotic diffusion may take place on regular trajectories. On the other hand, on some chaotic trajectories disorder induces regular motion. A localization effect similar to the Golosov phenomenon sets in whenever a disorder threshold that depends on the mass of the particle is reached. Possible applications of the localization phenomenon are discussed.

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Thermal ratchets [1] are simple stochastic models where a nonzero net drift speed may be obtained from time correlated fluctuations interacting with asymmetric periodic structures. The study of ratchets has received much attention due to their general interest in modeling molecular motors [2]. Another sources of interest is the possible application of ratchets for modeling nanoscale friction [3], the potential for the development of new approaches for separation of microscopic and mesoscopic objects [4], models for understanding surface smoothening [5], and the building of micron-scale devices [6].

In thermal ratchets, thermal fluctuations are rectified in different ways according to the type of ratchet system. For example, in the *rocking ratchet*, which is the most common type of ratchet, a time-dependent external driving force of zero average acts as the rectifier pumping mechanism. In this kind of ratchet, thermal noise does indeed help the ratchet by increasing its efficiency [7]; in contrast, spatial disorder reduces the characteristic drift speed [8,9] of thermal ratchets.

Recently, the influence of quenched disorder, in the absence of noise, on a periodically forced overdamped particle in a periodic asymmetric potential was considered [10]. An outcome of this study was the discovery of diffusive transport in the presence of quenched disorder. Diffusion was observed even with small amounts of added disorder and the diffusion current was found to increase with increasing noise and eventually reach the same order of magnitude as the regular drift. As is common in the study of thermal ratchets, this study was carried out in the Smoluchowski limit of vanishing mass. However, inertial effects are important in many experimental situations. For example, the finite mass of the particles plays an important role in friction at the nanoscale as well as in microscopic particle separation experiments. Thus, it would be of interest to study the effects of quenched disorder on the dynamics of underdamped thermal ratchets with finite mass.

Inertial ratchets, even in the absence of noise, have a very complex dynamics, including chaotic motion [11,12]. This deterministically induced chaos mimics, to some extent, the role of noise [13]. This added complexity drastically changes some of the basic properties of thermal ratchets. For example, it has been shown that inertial ratchets can exhibit multiple reversals in the current direction [11,12]. It has been suggested [12] that this behavior may be related to a crisis in which a chaotic ratchet state suddenly becomes periodic, but it was shown later [14] that current reversals can occur even in the absence of bifurcations from chaotic to periodic motion.

The aim of the present paper is to study the effects of spatial disorder on the dynamics of the underdamped deterministic ratchet, especially the influence of disorder on regular and chaotic motion and current reversals. We will concentrate on the model of a particle of nonvanishing mass, periodically driven in an asymmetric periodic potential with quenched disorder. No temporal noise term is considered, just the quenched disorder.

In scaled nondimensional coordinates, the equation of motion is given by [11]

$$\epsilon \frac{d^2x}{dt^2} + \gamma \frac{dx}{dt} = \cos(x) + \mu \cos(2x) + \Gamma \sin(\omega t) + \alpha \xi(x). \tag{1}$$

Here, ϵ is the mass of the particle, γ is the damping coefficient, Γ and ω are, respectively, the amplitude and frequency of an external oscillatory forcing, and $\alpha\xi(x)$ is the term due to the quenched disorder. The terms $\xi(x)$ are independent, uniformly distributed random variables with no spatial correlations, corresponding to a piecewise constant force on the period of the potential. The coefficient $\alpha \geqslant 0$ is the strength of the quenched disorder. The unperturbed ratchet potential,

$$U(x) = -\sin(x) - \mu \sin(2x), \tag{2}$$

has been the subject of extensive recent studies [10,11,15,16] mainly in models with no disorder. A recent work [12] has analyzed the influence of the chaotic behavior in Eq. (1) with $\alpha = 0$ and has related it, with the help of a bifurcation diagram, to the observed reversals in flow direction, but an *a posteriori* study has reexamined some of its conclusions [14]. Here we consider the addition of quenched disorder in order to analyze the effect of a more realistic representation of the substrate on the dynamics of the ratchet. We note that with the disorder term included, the ratchet equation (1) can

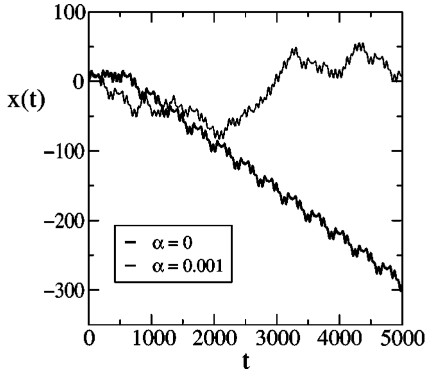

FIG. 1. Trajectory of the particle for $\Gamma=0.9245$, $\epsilon=20$, $\gamma=1.0$, $\mu=0.25$, $\omega=0.1$ with no quenched disorder (thick line) and with a very small amount, $\alpha=0.001$, of quenched disorder (thin line).

also be used to model fluctuations in dc current amplitude in arrays of Josephson junctions [17] and in studies of friction, particularly the sliding motion of clusters on surfaces [3].

Previous work [11] has shown that in the absence of quenched disorder ($\alpha=0$) there are both regular and chaotic solutions of Eq. (1) as well as multiple current reversals—arguably related to crisis in the underlying dynamics [12,14]. In the present work, we study the influence of quenched disorder on the system for both kinds of trajectories (regular and chaotic). Specifically, numerical solutions of Eq. (1) were obtained using a variable step Runge-Kutta-Fehlberg method [18]. We let $\epsilon=20$, $\gamma=1.0$, $\mu=0.25$, $\omega=0.1$, and studied the behavior for several values of Γ (see below). The calculational details are the same as in [10].

In Fig. 1, we show a typical periodic trajectory at $\Gamma=0.9245$ in the absence of disorder (the thick line) and the corresponding trajectory with a very small amount of quenched noise, $\alpha=0.001$ (the thin line). It is apparent from the figure that the trajectory gets modified. Figure 2 shows the corresponding phase portrait confined to the x interval $(-2\pi,0]$. Notice that we exhibit in the same plot both the case with no disorder (the points at the center of the six squares) and the chaotic attractor that it becomes after the quenched noise is added. The phase portrait confined to the same x interval is valid with disorder because the small disorder term may be considered as a small perturbation.

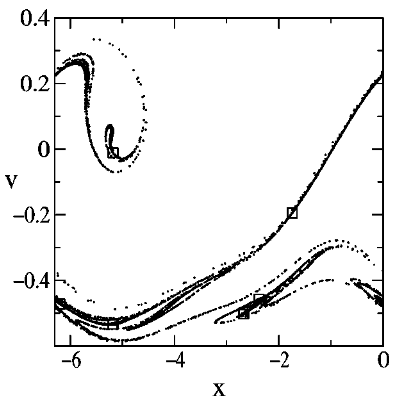

FIG. 2. Phase portrait of two attractors of the ratchet equation for $\Gamma=0.9245$, $\epsilon=20$, $\gamma=1.0$, $\mu=0.25$, $\omega=0.1$. With no quenched noise, the attractor consists of the points at the center of the squares. With a very small amount, $\alpha=0.001$, of quenched disorder the attractor becomes chaotic.

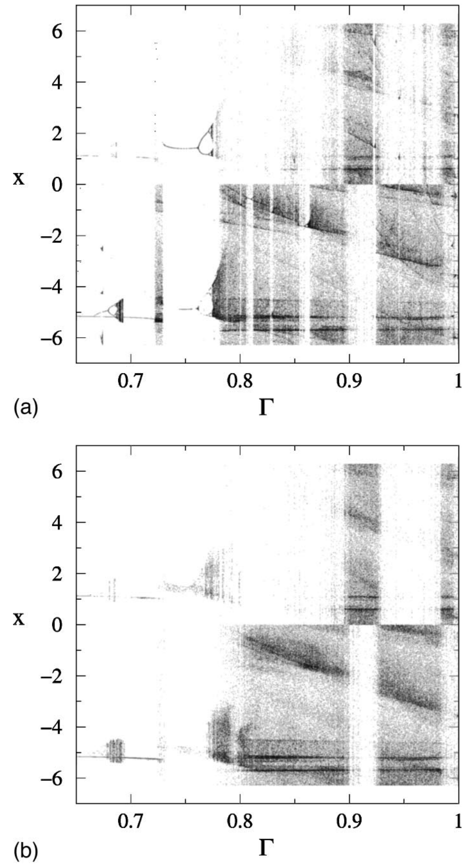

FIG. 3. (a) Bifurcation diagram as a function of Γ for $\epsilon=20$, $\gamma=1.0$, $\mu=0.25$, $\omega=0.1$ in the no-quenched-disorder case. (b) Bifurcation diagram as a function of Γ for $\epsilon=20$, $\gamma=1.0$, $\mu=0.25$, $\omega=0.1$ with an amount, $\alpha=0.001$, of quenched disorder.

In order to get a global picture of the behavior, in Fig. 3 we show the bifurcation diagram of Eq. (1) as a function of $\Gamma\in[0.65,1)$, both in the case with no disorder [Fig. 3(a)] and with a small, $\alpha=0.01$, quantity of quenched noise added [Fig. 3(b)]. The chosen range for Γ corresponds to the existence of regular and chaotic solutions and inversion of current in the absence of disorder. In all the chaotic cases analyzed, the sign of the current for a given Γ coincides with the sign of the majority of x of the corresponding trajectory in the bifurcation diagram; for the periodic states, on the other hand, we could not find any definitive correlation between the current and the bifurcation behavior.

Figure 3(b) also shows the different ways in which deterministic states are affected when quenched disorder is introduced. For example, for $\Gamma\in(0.8,0.9)$ (see also Fig. 2), states that are periodic in the deterministic case are changed to chaotic states. Consider a periodic state within a thin zone in the bifurcation diagram [as in the Γ zone roughly between 0.854 and 0.864 in Fig. 3(a)]. The effect of the quenched noise on the system is to send the ratchet to a nearby chaotic zone, as should be clear from Figs. 3(a) and 3(b).

On the other hand, some irregular states in the region around $\Gamma \simeq 0.73$ of Fig. 3 are changed to more regular states. This last effect can be associated with the taming of chaos with disorder [19], where disorder induces ordered motion characterized by very complex but nevertheless regular patterns. In a wide periodic window (like the one roughly between 0.693 and 0.722), the quenched noise has little effect on the periodic behavior, except near its upper end, where the chaotic zone itself becomes wider and superposes itself on the originally regular zone. Such behavior can also be seen by comparing Fig. 3(a) with Fig. 3(b). The main point then is that the ratchet dynamics follows the dominant behavior (of the Γ zone it belongs to) when a small amount of quenched noise is added.

In a recent paper, Popescu *et al.* [10] have shown that quenched disorder induces a significant additional chaotic ''diffusive'' motion on the overdamped version of Eq. (1). Thus, in the finite inertia case we are considering, strong fluctuations are expected and this calls for the use of a time-dependent probability measure, as has been previously done in [10,11].

A Gaussian distribution was chosen as the initial probability density. Apart from early transients, a linear mean and a linear variance—which are characteristics of a Brownian motion—are observed. The third- and higher-order cumulants increase slower than $t^{n/2}$, up to $n=6$, with n the order of the cumulant. Therefore, the probability distribution $p_t(x)$ is asymptotically a Gaussian, and, as is well known, the first and second cumulants, associated with the mean and the variance, are sufficient to describe the asymptotic evolution of $p_t(x)$ [10,11].

Averages were performed over ensembles of 5000 trajectories starting from different initial conditions very close to the origin $x=0$. The ensemble described above was left to evolve for 800 external drive periods T, and every 10 periods the positions $x(t)$ were stored for further analysis.

We first consider the case of periodic behavior with $\Gamma=0.9245$. In Figs. 4(a) and 4(b) we show results for the first and second moments, i.e., $C_1(t)=\langle x(t)\rangle$ and $C_2(t)=\langle[x(t)-\langle x(t)\rangle]^2\rangle$, respectively, where $\langle\cdots\rangle$ means the average over the ensemble, as a function of time t without quenched disorder ($\alpha=0$), and with two different small amounts of quenched disorder ($\alpha=0.005$ and $\alpha=0.01$). For $\alpha=0$, $C_1(t)\simeq v(\alpha)t$ and $C_2(t)$ tends to a constant as $t\to\infty$, which characterizes a periodic state. For $\alpha=0.005$ and $\alpha=0.01$, both first and second moments show an asymptotic linear dependence on time t, $C_1(t)\simeq v(\alpha)t$, $C_2(t)\simeq D(\alpha)t$. There is a reversal of current due to the presence of quenched disorder, and at the same time the onset of diffusion. This last effect is also obtained by adding a small amount of quenched disorder to the overdamped ratchet [10].

To study the effect of quenched disorder on a chaotic trajectory, we now consider the case of $\Gamma=0.8967$. We calculate the first and second moments, $C_1(t)$ and $C_2(t)$, both with no quenched disorder ($\alpha=0$) and with different small amounts of quenched disorder ($\alpha=0.005$, $\alpha=0.01$, and $\alpha=0.05$). The corresponding Figs. 5(a) and 5(b) show that again there is a current reversal and the magnitude of the current increases with increasing quenched disorder. The existing diffusive behavior is also slightly increased.

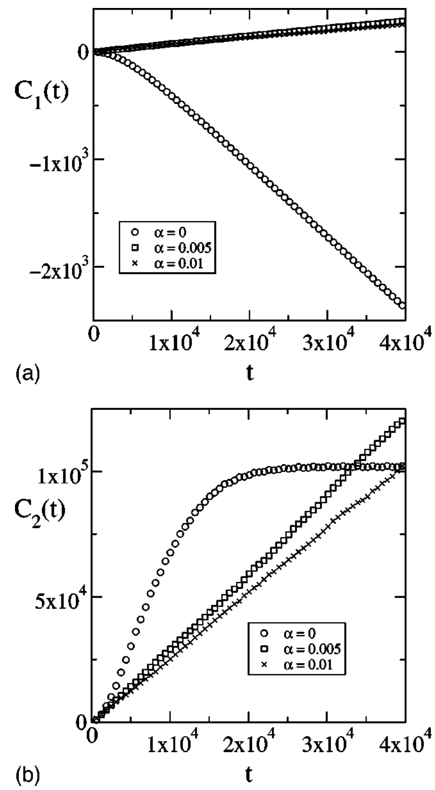

FIG. 4. Influence of disorder on the probability distribution for $\Gamma=0.9245$, regular with no disorder for three different amounts of quenched disorder; $\alpha=0$, $\alpha=0.005$, and $\alpha=0.01$: (a) First moment $C_1(t)$ for $\epsilon=20$, $\gamma=1.0$, $\mu=0.25$, $\omega=0.1$ as a function of time. (b) Second moment $C_2(t)$ for $\epsilon=20$, $\gamma=1.0$, $\mu=0.25$, $\omega=0.1$ as a function of time.

It is interesting to note that the reversal of the current is not always associated with the addition of quenched disorder, as can be seen in Figs. 6(a) and 6(b), where we have plotted $C_1(t)$ and $C_2(t)$ for the chaotic trajectory corresponding to $\Gamma=0.8955$ without quenched disorder ($\alpha=0$), and with different amounts of quenched disorder ($\alpha=0.03$, $\alpha=0.05$, and $\alpha_t\simeq 0.1$). The current maintains its direction and increases with increasing disorder until $\alpha=0.1$. With further increase in α, the current goes to zero asymptotically. Diffusion also increases with increasing disorder for $\alpha<0.1$, but it tends to zero asymptotically with increasing α. This localization effect may be clearly seen in the particle trajectories when the particle gets stuck or oscillates with the same amplitude for several periods.

A localization effect in random walks on random environments is known as the Golosov phenomenon in the random-walk literature [20]. Its occurrence has been proven rigorously for systems with only nearest-neighbor transitions by Golosov [21]. It was also reported by Radons [22] on one-dimensional chaotic maps where chaotic diffusion is totally suppressed by the presence of quenched disorder. The Golosov phenomenon may be described as a packet of initially close particles moving in a coherent fashion from one minimum to the next deeper minimum. Hence, it may also occur in a disordered ratchet when and if particle motion

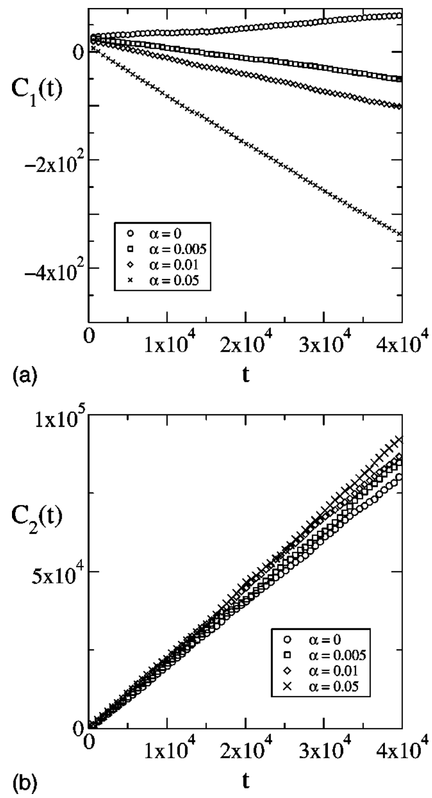

FIG. 5. Influence of disorder on the probability distribution for $\Gamma=0.8967$, chaotic with no disorder for four different amounts of quenched disorder; $\alpha=0$, $\alpha=0.005$, $\alpha=0.01$, and $\alpha=0.05$: (a) First moment $C_1(t)$ for $\epsilon=20$, $\gamma=1.0$, $\mu=0.25$, $\omega=0.1$ as a function of time. (b) Second moment $C_2(t)$ for $\epsilon=20$, $\gamma=1.0$, $\mu=0.25$, $\omega=0.1$ as a function of time.

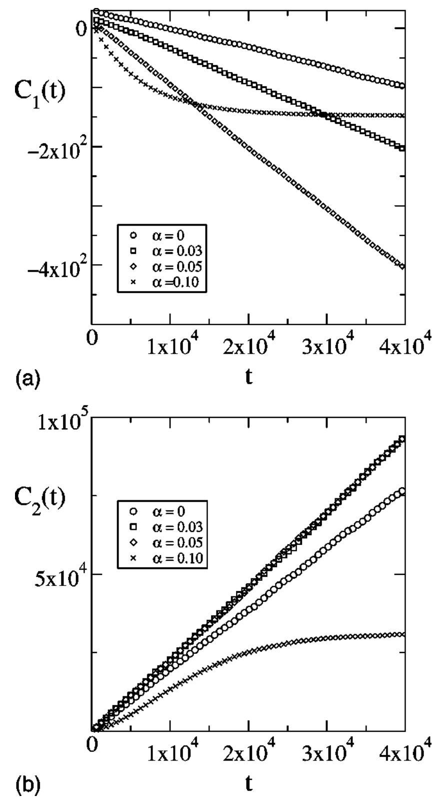

FIG. 6. Influence of disorder on the probability distribution for $\Gamma=0.8955$, chaotic with no disorder. Diffusion and current increase keeping direction of current until localization sets in. Four different amounts of quenched disorder are shown: $\alpha=0$, $\alpha=0.03$, $\alpha=0.05$, and $\alpha=0.10$. At $\alpha=0.1$ current and diffusion vanish asymptotically, which means localization. (a) First moment $C_1(t)$ for $\epsilon=20$, $\gamma=1.0$, $\mu=0.25$, $\omega=0.1$ as a function of time. (b) Second moment $C_2(t)$ for $\epsilon=20$, $\gamma=1.0$, $\mu=0.25$, $\omega=0.1$ as a function of time.

becomes locked to the external driving frequency. Our results show that for all values of Γ studied, current and diffusion asymptotically tend to zero if the amount of quenched disorder exceeds a certain threshold. It is important to note that according to our preliminary results, this threshold depends on the mass of the particle, it being smaller when the mass of the particle decreases. This is clearly an instance of a localization effect analogous to the Golosov phenomenon.

An important application of the localization effect may be in microscopic particle separation [4]. The reason is that the threshold value of quenched disorder for the appearance of the localization effect seems to depend on the mass of the particle. Thus, it may be possible to develop a new approach for separating microscopic particles without having to provide an applied gradient. This can be accomplished by tuning the amount of quenched disorder on a microfabricated sieve in a way that particles of smaller mass, with lower disorder threshold, remain stacked while more massive particles and with higher disorder threshold display a net current.

In summary, we have studied the effects of small amounts of quenched disorder on an underdamped (inertial) ratchet. In analogy with the case of overdamped ratchets, we found that strong diffusive motion may be induced in the periodic trajectories. On the other hand, we found that for some values of Γ associated with chaotic trajectories in the overdamped case, quenched disorder induces regular solutions. Disorder can cause current reversals in both chaotic and regular solutions, and whenever the amount of quenched disorder exceeds a certain threshold the motion becomes localized.

These results should be helpful in the interpretation of experimental results in studies of friction, particularly at the nanoscale, as well as in understanding transport processes in molecular motors. An interesting direction for future research may be to study the dependence of the localization phenomenon on the particle mass. This question is of interest due to its possible application in designing new particle separation techniques.

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